Phase 13A – Mathematical Consolidation  
Part 3: Reformulating ψ Dynamics into a Unified Action

Goal  
I assemble a unified variational action that encodes the ψ substrate, the effective potential Φ, and the current while preserving the core ψ-gravity link

Plain-text:  
G(x) = (∇²Φ) ψ(x)

All symbolic algebra and consistency checks below are performed by the AI; I present the results in first-person voice as the project author.

Unified Action (covariant form)  
I propose the covariant action

Plain-text:  
S[ψ,J\_μ,S] = ∫ d⁴x √(-g) [ ½ K(Φ) g^{μν} ∇\_μψ ∇\_νψ + ½ (∇²Φ) ψ² − α/2 (J\_μ J^μ) ψ² − V(ψ,Φ) ]

Notes:

* K(Φ) is a medium-response prefactor (symbolic function).
* α is a coupling constant controlling direct J²ψ² interaction.
* V(ψ,Φ) captures additional self- and Φ-dependent potentials.
* Φ(x) = S(x) + g^{μν} J\_μ J\_ν.

I will use the first-derivative equivalent (integration by parts) when convenient to avoid higher-derivative ambiguities in Noether/Euler–Lagrange manipulations.

First-derivative equivalent Lagrangian (used for variations)

Integrating the ½(∇²Φ)ψ² term by parts (dropping a boundary term) gives the dynamically equivalent density

Plain-text:  
L = ½ K(Φ) g^{μν} ∇\_μψ ∇\_νψ − ½ g^{μν} (∇\_μΦ) ∇\_ν(ψ²) − α/2 (J\_μ J^μ) ψ² − V(ψ,Φ)

Equation of Motion — variation with respect to ψ

The Euler–Lagrange equation for ψ reads

Plain-text:  
∇^μ( K(Φ) ∇\_μ ψ ) + (∇²Φ) ψ − α (J\_μ J^μ) ψ − ∂V/∂ψ = 0

Remarks:

* When K(Φ) = 1 this reduces to □ψ + (∇²Φ)ψ − α(J²)ψ − ∂\_ψV = 0, with □ ≡ ∇^μ ∇\_μ.
* The (∇²Φ)ψ term explicitly reinstates the core ψ-gravity coupling.

Variation with respect to Φ (constraint equation)

Treating Φ as a dynamical/composite field in the action (via K(Φ) and V), the Euler–Lagrange equation for Φ is

or equivalently

Plain-text:  
½ K’(Φ) (∇ψ)² − ∂\_Φ V(ψ,Φ) + ½ ∇²(ψ²) = 0  
or: ∇²(ψ²) + K’(Φ) (∇ψ)² − 2 ∂\_Φ V(ψ,Φ) = 0

Interpretation:

* This equation links spatial structure of ψ (through ∇²(ψ²)) to how medium-response K and potential V depend on Φ.
* Since Φ contains J², this is the channel where current-structure feeds back onto ψ-distributions.

Variation with respect to J\_μ (current equation / constraint)

Because Φ = S + J² and J² = , the Euler–Lagrange equation for J\_μ can be written as

Plain-text:  
δS/δJ\_μ = 2 J^μ [ ½ K’(Φ) (∇ψ)² − ∂\_Φ V(ψ,Φ) + ½ ∇²(ψ²) ] − α ψ² J^μ = 0

Hence a compact factorized form:

Plain-text:  
J^μ [ ∇²(ψ²) + K’(Φ) (∇ψ)² − 2 ∂\_Φ V(ψ,Φ) − α ψ² ] = 0

Two branches:

* Either J^μ = 0 (current-free branch), or
* The bracket vanishes, imposing a nontrivial constraint coupling ψ, V, and Φ.

Canonical momentum and Hamiltonian density (field-level)

Canonical momentum conjugate to ψ:

Plain-text:  
π\_ψ = ∂L / ∂(∂\_t ψ) = K(Φ) g^{tν} ∇\_ν ψ − g^{μt} (∇\_μ Φ) ψ

Hamiltonian density:

Plain-text:  
H\_ψ = π\_ψ ∂\_t ψ − L

In explicit form:

Plain-text:  
H\_ψ = ½ π\_ψ² + ½ K(Φ) (∇ψ)² − ½ (∇²Φ) ψ² + α/2 (J\_μ J^μ) ψ² + V(ψ,Φ) + (mixing terms from ∇Φ·∇(ψ²))

Stability / Physical consistency comments

* The action is second-order in derivatives for the basic fields when kept in the original form; I use the integrated-by-parts first-derivative form for safe variational calculus.
* K(Φ) must be positive-definite to avoid ghost instabilities in ψ.
* The α term provides a straightforward channel for current-driven effective mass:
* The Φ-variation equation shows how Laplacian coupling translates into a source-like constraint feeding back to Φ (hence to currents and space).

Python Symbolic Prototype

# simulations/phase13A\_part3\_unified\_action.py  
import sympy as sp  
  
# Coordinates and flat metric (Minkowski signature) for prototype  
t, x, y, z = sp.symbols('t x y z')  
coords = (t, x, y, z)  
eta = sp.diag(-1, 1, 1, 1) # g^{μν}  
  
# Fields (symbolic functions of coordinates)  
psi = sp.Function('psi')(\*coords) # ψ(x)  
S = sp.Function('S')(\*coords) # space(x)  
J = [sp.Function(f'J{mu}')(\*coords) for mu in range(4)] # J\_μ(x)  
  
# Parameters / functions  
alpha = sp.symbols('alpha') # coupling constant  
K = sp.Function('K') # K(Φ)  
V = sp.Function('V') # V(ψ, Φ)  
  
# Current squared: J^2 = g^{μν} J\_μ J\_ν  
current\_sq = sum(eta[mu,nu]\*J[mu]\*J[nu] for mu in range(4) for nu in range(4))  
  
# Effective potential Φ  
Phi = S + current\_sq  
  
# Gradients  
grad\_psi = [sp.diff(psi, c) for c in coords]  
grad\_Phi = [sp.diff(Phi, c) for c in coords]  
  
# (∇ψ)^2 and ∇^2(ψ^2) (flat prototype)  
grad\_psi\_sq = sum(eta[mu,nu]\*grad\_psi[mu]\*grad\_psi[nu] for mu in range(4) for nu in range(4))  
laplacian\_psi2 = sum(eta[mu,nu]\*sp.diff(psi\*\*2, coords[mu], coords[nu]) for mu in range(4) for nu in range(4))  
  
# First-derivative equivalent coupling: −1/2 g^{μν} (∂\_μΦ) ∂\_ν(ψ^2)  
coupling\_first\_deriv = -sp.Rational(1,2) \* sum(eta[mu,nu]\*grad\_Phi[mu]\*sp.diff(psi\*\*2, coords[nu]) for mu in range(4) for nu in range(4))  
  
# Lagrangian density (first-deriv form):  
LPhi = sp.Rational(1,2) \* K(Phi) \* grad\_psi\_sq + coupling\_first\_deriv - sp.Rational(1,2) \* alpha \* current\_sq \* psi\*\*2 - V(psi, Phi)  
  
# Euler-Lagrange for ψ  
EL\_psi = sum(sp.diff(sp.diff(LPhi, sp.diff(psi, coords[mu])), coords[mu]) for mu in range(4)) - sp.diff(LPhi, psi)  
EL\_psi\_simpl = sp.simplify(EL\_psi)  
  
# Euler-Lagrange for Φ  
EL\_Phi = sum(sp.diff(sp.diff(LPhi, sp.diff(Phi, coords[mu])), coords[mu]) for mu in range(4)) - sp.diff(LPhi, Phi)  
EL\_Phi\_simpl = sp.simplify(EL\_Phi)  
  
# Euler-Lagrange for J\_μ (δΦ/δJ\_μ = 2 J^μ)  
dS\_dPhi = EL\_Phi\_simpl  
dS\_dJ = [sp.simplify(2\*J[mu]\*dS\_dPhi - alpha \* psi\*\*2 \* J[mu]) for mu in range(4)]  
  
# Canonical momentum π\_ψ = ∂L/∂(∂\_t ψ)  
pi\_psi = sp.diff(LPhi, sp.diff(psi, t))  
  
# Hamiltonian density H = π\_ψ ∂\_t ψ − L  
H\_psi = sp.simplify(pi\_psi\*sp.diff(psi, t) - LPhi)  
  
# Print key symbolic expressions  
print("Φ (effective) =", Phi)  
print("Lagrangian density L =", LPhi)  
print("Euler-Lagrange (ψ) =", EL\_psi\_simpl)  
print("Euler-Lagrange (Φ) =", EL\_Phi\_simpl)  
print("δS/δJ\_μ (prototype) =", dS\_dJ)  
print("π\_ψ =", pi\_psi)  
print("Hamiltonian density H\_ψ =", H\_psi)